

Forecasting the VIX in the midst of COVID-19

Supplementary Information

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Abstract

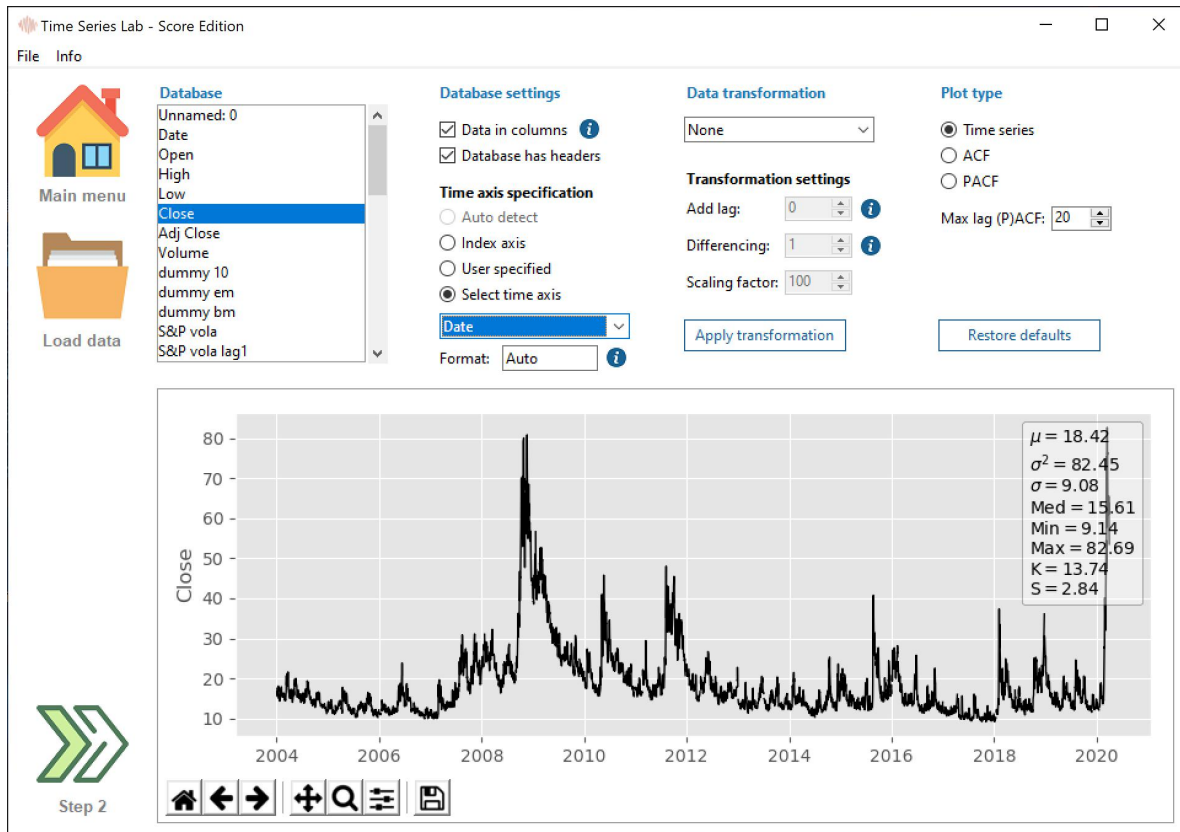
We study the behavior of the Volatility Index (VIX) time series in the period leading up to the COVID-19 outbreak. Time-varying location/scale models are used to extract a range of unobserved components from the VIX time series. The time-varying unobserved components are driven by the score of the predictive density. These so called score-driven models have proven to be powerful in extracting unobserved components like autoregressive processes and seasonal patterns. A range of model specifications is used to forecast the VIX in the COVID-19 period that spans the first quarter of 2020. Explanatory variables are used to improve in-sample model fit and out-of-sample forecast accuracy. All model computations are carried out with the *Time Series Lab* software package.

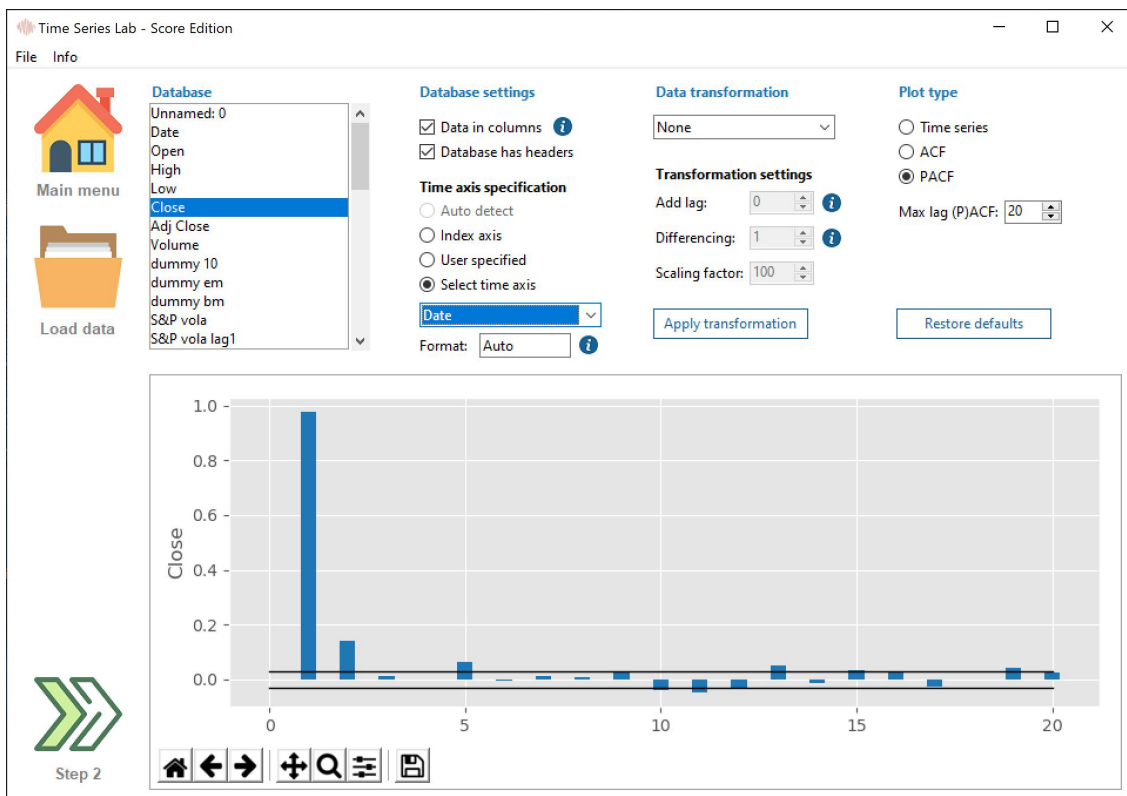
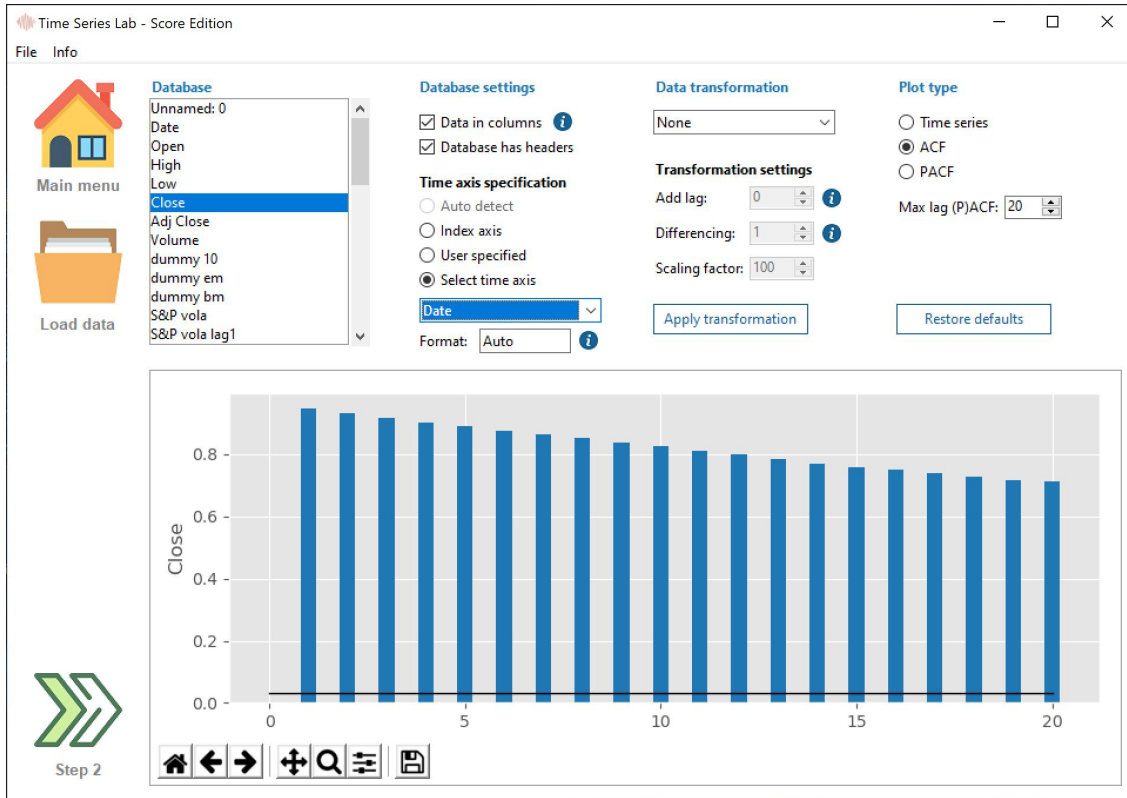
Key words: VIX, COVID-19, Time Series Lab, Unobserved components, Time Series, Forecasting, Score-driven models

Five step procedure

The *Time Series Lab* modelling and forecasting of time series is a five-step procedure. For basic models, not many choices need to be made while for others some advanced settings need to be adjusted. We present screenshots of *Time Series Lab* that were made during the research that lead to the results in this paper.


Step 1: Load data






Step 2: Model setup

Time Series Lab - Score Edition
File Info



Main menu



Advanced settings

Distribution

Select dependent variable
Close

Distribution group

Continuous
 Discrete

Continuous distributions

Gaussian
 Student t
 Generalized Error (GED)
 Exp. Generalized Beta 2
 Exponential

Select components for location

Static location

Dynamic components

Level

Random walk
 Random walk + slope
 Integrated random walk

Autoregressive I of order 2

Autoregressive II of order 1

Seasonal length 5

Explanatory variables

Adjust variable selection

Select components for scale

Static scale

Dynamic components

Level

Random walk
 Random walk + slope
 Integrated random walk

Autoregressive I of order 1

Autoregressive II of order 1

Seasonal length 4

Leverage effect

Explanatory variables

Adjust variable selection

Model specification

Distribution
The dependent variable is Close
The selected distribution is the Student t distribution with support $y \in [+- \text{Inf}]$
The Student t distribution has the following parameters:


Parameters	Symbol	Time-varying	Domain
Mean	μ	Yes	+/- Inf
Scale	σ	Yes	> 0
Degrees of freedom	ν	No	> 2

Parameter specification
 $\mu = \text{exp}(\text{AR}(2) + \text{AR}(1) + \text{Seasonal} + X\beta)$
 $\sigma = \text{exp}(\text{AR}(1))$
 $\nu = \text{constant}$


Explanatory variables
Explanatory variables for location are: dummy em | dummy bm | S&P neg return lag1 | WTI Close lag1

Initialisation of location
Initialisation component: Autoregressive cmp I
Type of initialisation: Estimate

Initialisation of scale
Initialisation component: Autoregressive cmp I
Type of initialisation: Unconditional mean




Step 1




Step 3

Time Series Lab - Score Edition
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File Info




Main menu



Distribution



Step 1



Step 3

Score scaling ⓘ

Location

Unit (no scaling)

Inverse Fisher

Inverse square root Fisher

Score lags:

Scale

Unit (no scaling)

Inverse Fisher

Inverse square root Fisher

Score lags:

Advanced settings location

Initialisation component

Level

Autoregressive I

Autoregressive II

Type of initialisation

Unconditional mean

Estimate

Log mean of data sample

sample range: 1 -

Type of link function

Unit

Exponential

Logit

Advanced settings scale

Initialisation component

Level

Autoregressive I

Autoregressive II

Type of initialisation

Unconditional mean

Estimate

Log variance of data sample

sample range: 1 -

Type of link function

Unit

Exponential

Logit

Model specification

Distribution

The dependent variable is Close

The selected distribution is the Student t distribution with support $y \in [+/- \text{Inf}]$

The Student t distribution has the following parameters:

Parameters	Symbol	Time-varying	Domain
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Parameter specification

$\mu = \exp(\text{AR}(2) + \text{AR}(1) + \text{Seasonal} + X\beta)$

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Explanatory variables

Explanatory variables for location are: dummy em | dummy bm | S&P neg return lag1 | WTI Close lag1

Initialisation of location

Initialisation component: Autoregressive cmp I

Type of initialisation: Estimate

Initialisation of scale

Initialisation component: Autoregressive cmp I


Type of initialisation: Unconditional mean

5


Step 3: Estimation

Time Series Lab - Score Edition
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File Info




Main menu



Estimate



Step 2



Step 4

Edit and fix parameter values

Set defaults
Set estimates

	Parameter	Value	In bounds
<input type="checkbox"/>	Log location: AR2 ω	0.14565982	✓
<input type="checkbox"/>	Log location: AR2 α	0.02	✓
<input type="checkbox"/>	Log location: AR2 ϕ_1	0.85	✓
<input type="checkbox"/>	Log location: AR2 ϕ_2	0.1	✓
<input type="checkbox"/>	Log location: init	2.9132	✓
<input type="checkbox"/>	Log location: AR1 (2nd) α	0.02	✓
<input type="checkbox"/>	Log location: AR1 (2nd) ϕ	0.7	✓
<input checked="" type="checkbox"/>	Log location: seasonal α	0	✓
<input type="checkbox"/>	Log location: init seasonal 1	0	✓
<input type="checkbox"/>	Log location: init seasonal 2	0	✓
<input checked="" type="checkbox"/>	Log location: init seasonal 3	0	✓
<input checked="" type="checkbox"/>	Log location: init seasonal 4	0	✓
<input type="checkbox"/>	Log location: β _dummy em	0.0	✓
<input type="checkbox"/>	Log location: β _dummy bm	0.0	✓
<input type="checkbox"/>	Log location: β _S&P neg return la..	0.0	✓
<input type="checkbox"/>	Log location: β _WTI Close lag1	0.0	✓
<input type="checkbox"/>	Log scale: AR1 ω	0.09753459	✓
<input type="checkbox"/>	Log scale: AR1 α	0.02	✓
<input type="checkbox"/>	Log scale: AR1 ϕ	0.95	✓
<input type="checkbox"/>	Degrees of freedom	5.0	✓

Estimation options

Select estimation method

Maximum Likelihood (BFGS, numerical score) ⓘ

No estimation

Estimation sample

Estimation starts at t =

Estimation ends at t =

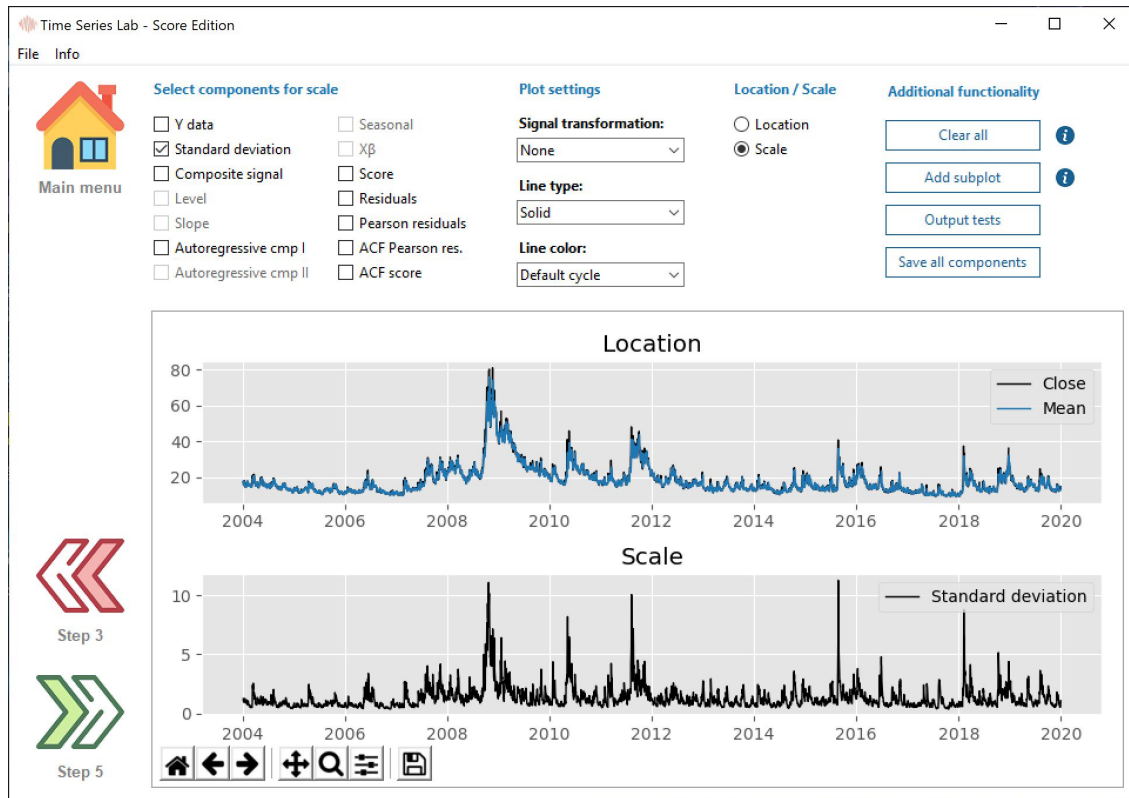
Additional settings

Print output every i'th iteration

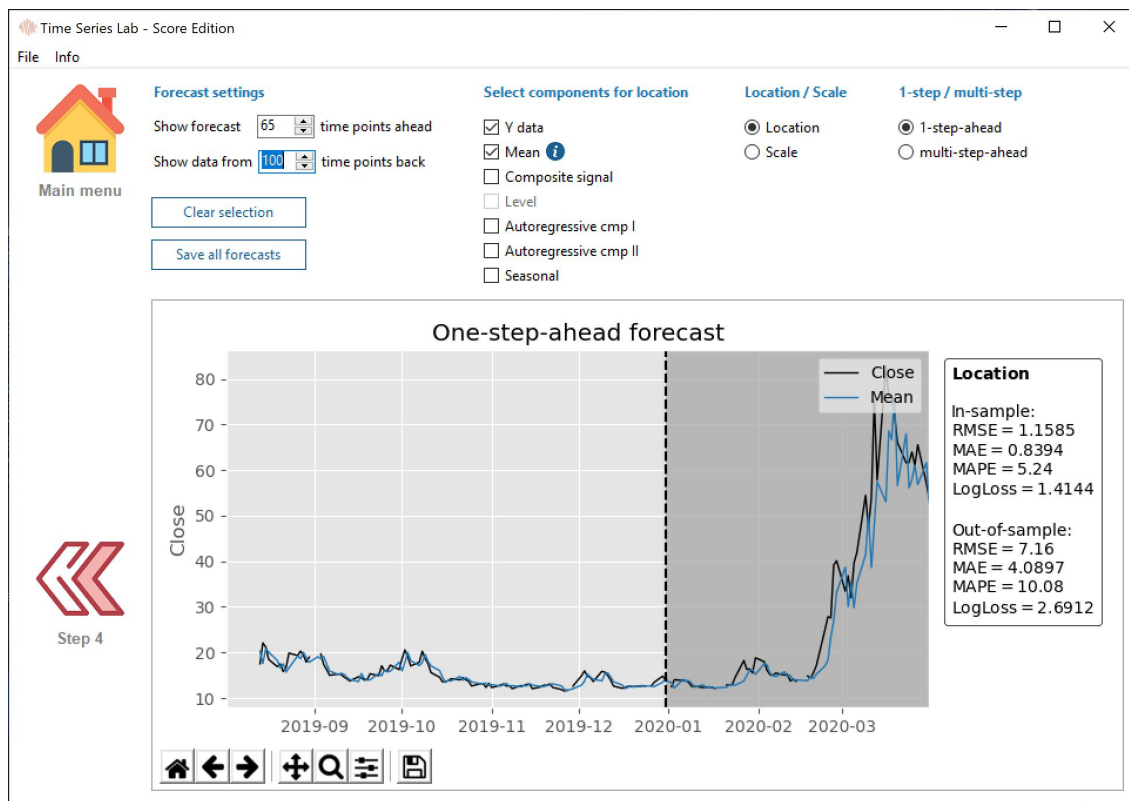
Additional output

Parameter report

Step 4: Graphics




Step 5: Forecasting





Output


Time Series Lab - Score Edition


File Info

 Load data

 Model setup

 Estimation

 Graphics

 Forecasting

```

it20  t= -1.47700637
it30  f= -1.47700339
it40  f= -1.47700295
it50  f= -1.47700279
it60  f= -1.47700170
it70  f= -1.47699985
it80  f= -1.47699821
it85  f= -1.47699821
                    
```

Strong convergence using numerical derivatives
Log-likelihood = -6162.036530; T = 4172

Optimized parameter values:

Parameter type	Value	Free/Fix
Log location: AR2 ω	0.0091	Free
Log location: AR2 α	0.3278	Free
Log location: AR2 ϕ_1	1.0073	Free
Log location: AR2 ϕ_2	-0.0113	Free
Log location: init	2.8509	Free
Log location: AR1 (2nd) α	0.4899	Free
Log location: AR1 (2nd) ϕ	0.8628	Free
Log location: seasonal α	0.0000	Fixed
Log location: init seasonal 1	0.0075	Free
Log location: init seasonal 2	0.0050	Free
Log location: init seasonal 3	0.0000	Fixed
Log location: init seasonal 4	0.0000	Fixed
Log location: β_{dummy} em	0.0184	Free
Log location: β_{dummy} bm	0.0084	Free
Log location: $\beta_{S\&P}$ neg return la..	-0.3429	Free
Log location: β_{WTI} Close lag1	-0.2426	Free
Log scale: AR1 ω	0.0052	Free
Log scale: AR1 α	0.1536	Free
Log scale: AR1 ϕ	0.9259	Free
Degrees of freedom	7.4019	Free

Estimation process completed in 361.6318 seconds

STATE INFORMATION

Component location	Initial	Time T
Mean	17.4334	14.4714
Composite signal	2.8584	2.6722
AR2	2.8509	2.5987
AR1 (2nd)	0.0000	0.0640
Seasonal	0.0075	0.0050
X β	0.0000	0.0044

Component scale	Initial	Time T
Standard deviation	1.2560	1.0926
Composite signal	0.0704	-0.0689
AR1	0.0704	-0.0689