Forecasting the VIX in the midst of COVID-19

Supplementary Information

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Abstract

We study the behavior of the Volatility Index (VIX) time series in the period leading up to the COVID-19 outbreak. Time-varying location/scale models are used to extract a range of unobserved components from the VIX time series. The time-varying unobserved components are driven by the score of the predictive density. These so called score-driven models have proven to be powerful in extracting unobserved components like autoregressive processes and seasonal patterns. A range of model specifications is used to forecast the VIX in the COVID-19 period that spans the first quarter of 2020. Explanatory variables are used to improve in-sample model fit and out-of-sample forecast accuracy. All model computations are carried out with the *Time Series Lab* software package.

Key words: VIX, COVID-19, Time Series Lab, Unobserved components, Time Series, Forecasting, Score-driven models

Five step procedure

The *Time Series Lab* modelling and forecasting of time series is a five-step procedure. For basic models, not many choices need to be made while for others some advanced settings need to be adjusted. We present screenshots of *Time Series Lab* that were made during the research that lead to the results in this paper.

Step 1: Load data







Step 2: Model setup

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|----------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------|--------------------------------------------------------------------------|----------------------------------|----|---|--|--|
| | | | | | | | | | |
| | Distribution | Select components for location | | Select components for scale | | | | | |
| | Select dependent variable | Static lo | ocation | | Static scale | | | | |
| | Close ~ | Dynamic c | omponents | | Dynamic components | | | | |
| menu | Distribution group | ictribution group | | | Level | | | | |
| - Contra | Continuous | Ra | indom walk | | Random walk | | | | |
| 0 | | O Ra | ndom walk + slope | | Random walk + slope | | | | |
| | Obsciele | | tegrated random wall | k | Integrated random walk | | | | |
| | Continuous distributions O Gaussian | Autoregressive I of order 2 | | | Autoregressive I of order 1 | | | | |
| ced | Student t | Autoreo | gressive II of order | 1 🚔 | 🗌 Autoregressive II of order 🕴 韋 | | | | |
| igs | Generalized Error (GED) | | al law ath 🖪 💌 🖉 | 9 | | | | | |
| | O Exp. Generalized Beta 2 | ✓ Season | al length 🖬 📼 | U | 🔄 Seasonal length 👻 🤿 😈 | | | | |
| | Exponential | 🗹 Explana | Explanatory variables | | 🗌 Leverage effect 🚺 | | | | |
| | | Adju | st variable selection | | Explanatory variables | | | | |
| | | | | | Adjust unvisible selection | | | | |
| | | | | | Adjust variable selection | | | | |
| | | | | | | | | | |
| | Model specification | | | | | | | | |
| | Model specification Distribution The dependent variable is Clos | e | | | ^ | • | | | |
| | Model specification Distribution The dependent variable is Clos The selected distribution is the | e Student t distrib | ution with support y | ∈ [+/- lnf] | ^ | • | | | |
| | Model specification Distribution The dependent variable is Clos The selected distribution is the The Student t distribution has t | e Student t distrib the following par | ution with support y ameters: | ∈ [+/- Inf] | ^ | • | | | |
| | Model specification Distribution The dependent variable is Clos The selected distribution is the The Student t distribution has t Parameters | e Student t distrib the following par Symbol | ution with support y ameters: Time-varying | ∈ [+/- Inf] Domain | ^ | • | | | |
| | Model specification Distribution The dependent variable is Clos The selected distribution is the The Student t distribution has t Parameters Mean Scale | e Student t distrib the following par Symbol ^µ | ution with support y ameters: Time-varying Yes Ves | € [+/- Inf] Domain +/- Inf | ^ | | | | |
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| 7 | $\label{eq:model} \begin{array}{l} \hline \textbf{Model specification} \\ \hline \textbf{Distribution} \\ The dependent variable is Clos \\ The selected distribution is the The Student t distribution has the Student t distribution has the selected scale \\ \hline \textbf{Parameters} \\ \hline \textbf{Mean} \\ Scale \\ \hline \textbf{Degrees of freedom} \\ \hline \textbf{Parameter specification} \\ \mu = \exp(AR(2) + AR(1) + Seaso \\ \sigma = \exp(AR(1)) \end{array}$ | e Student t distrib following par Symbol φ ν ν nal + Xβ) | ution with support y ameters: Time-varying Yes Yes No | € [+/- Inf] Domain +/- Inf > 0 > 2 | ^ | | | | |
| 7 | $\label{eq:second} \begin{array}{ c c c } \hline \textbf{Model specification} \\ \hline \textbf{Distribution} \\ \hline \textbf{The dependent variable is Clos} \\ \hline \textbf{The selected distribution is the} \\ \hline \textbf{The Student t distribution has t} \\ \hline \textbf{Parameters} \\ \hline \textbf{Mean} \\ \hline \textbf{Scale} \\ \hline \textbf{Degrees of freedom} \\ \hline \textbf{Parameter specification} \\ \mu = exp(AR(2) + AR(1) + Seaso \\ \sigma = exp(AR(1)) \\ v = constant \\ \end{array}$ | e Student t distrib the following par Symbol φ ν ν nal + Xβ) | ution with support y ameters: Time-varying Yes Yes No | ∈ [+/- Inf] Domain +/- Inf > 0 > 2 | ^ | | | | |
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| 2 | Model specification Distribution The dependent variable is Clos The selected distribution is the The Student t distribution has the Parameters Mean Scale Degrees of freedom Parameter specification $\mu = exp(AR(2) + AR(1) + Seaso$ $\sigma = exp(AR(2))$ $\nu = constant$ Explanatory variables Explanatory variables for locati | e Student t distrib the following par Symbol φ ν nal + Xβ) on are: dummy o | ution with support y ameters: Time-varying Yes Yes No | € [+/- Inf] Domain +/- Inf > 0 > 2 P neg return la | ng1 WTI Close lag1 | | | | |
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| «//н ті | ime Series Lab - | Score Edition | | | | | - | × |
|---------------------------------------------------|------------------|----------------------------------------------------------------------------------------------------|-------------------------------------------------------|--------------|-----------------------------|---|---|---|
| File | Info | | | | | | | |
| | | Score scaling | Advanced settings location | | Advanced settings scale | | | |
| | | Location | | n component | Initialisation component | | | |
| | | O Unit (no scaling) | Level | | Level | | | |
| | | Inverse Fisher | Autoregressive I | | Autoregressive I | | | |
| M | lain menu | O Inverse square root Fisher | ○ Autoreg | ressive II | ○ Autoregressive II | | | |
| j. | ···· • - | Score lags: 1 | Type of init | ialisation | Type of initialisation | | | |
| | :' / | | | | Unconditional mean | | | |
| - | d | Scale | Estimate | | O Estimate | | | |
| - Z | | O Unit (no scaling) | Log mean of data sample | | Log variance of data sample | | | |
| Di | istribution | Inverse Fisher | iverse Fisher sample range: 1 - 10 | | sample range: 1 - 10 🌲 | | | |
| | | O Inverse square root Fisher | | | | | | |
| | | Course Langer 1 | Type of link function Score lags: 1 | | Type of link function | | | |
| | | Score lags: | | | ⊖ Unit | | | |
| | | | Exponent | ntial | Exponential | | | |
| | | | O Logit | | O Logit | | | |
| | | | | | | | | |
| | | | | | | | | |
| | | Model specification | | | | | | |
| | | Distribution The dependent variable is Close | | | | | | |
| | | The selected distribution is the Stu | udent t distribution with support $y \in [+/- lnf]$ | | | | | |
| | | The Student t distribution has the | following par | ameters: | | | | |
| | | Parameters | Symbol | Time-varying | Domain | | | |
| | | Mean | μ | Yes | +/- Inf | | | |
| | | Scale Degrees of freedom | σ | Yes | > 0 | | | |
| | | Degrees of freedom V No > a | | | 7 E | | | |
| | | Parameter specification | | | | | | |
| | 77 | $\sigma = \exp(AR(2) + AR(1) + Seasonal\sigma = exp(AR(1))$ | + ^p) | | | | | |
| 1 | 11 | v = constant | | | | | | |
| | | Explanatory variables | | | | | | |
| | | Explanatory variables for location are: dummy em dummy bm S&P neg return lag1 WTI Close lag1 | | | | | | |
| | Step 1 | | | | | | | |
| - | _ | Initialisation of location Initialisation component: Autoregressive cmp I | | | | | | |
| | | Type of initialisation: Estimate | | | | | | |
| | //// | Initialisation of scale | | | | | | |
| | | Initialisation component: Autoregressive cmp I | | | | | | |
| Step 3 Type of initialisation: Unconditional mean | | | | | | 5 | | |
| | | | | | | • | | |

Time Series Lab - Score Edition × _ File Info Edit and fix parameter values **Estimation options** Set defaults Set estimates Select estimation method Maximum Likelihood (BFGS, numerical score) 1 O No estimation Fix Parameter Value In bounds Main menu Log location: AR2 ω 0.14565982 1 Estimation sample Estimation starts at t = 1 Log location: AR2 α 0.02 v CPU Estimation ends at t = 4172 Log location: AR2 ϕ_1 0.85 v Additional settings Log location: AR2 ϕ_2 0.1 ~ Print output every i'th iteration 10 Estimate Log location: init 2.9132 1 Log location: AR1 (2nd) α 0.02 V Additional output Log location: AR1 (2nd) ϕ 0.7 V Parameter report \checkmark Log location: seasonal α 0 1 0 1 Log location: init seasonal 1 0 Log location: init seasonal 2 V \checkmark Log location: init seasonal 3 0 v \checkmark Log location: init seasonal 4 0 1 Log location: β_dummy em 0.0 V Log location: β_{dummy} bm 0.0 V Log location: B_S&P neg return la.. 0.0 v Log location: **B_WTI** Close lag1 0.0 V Log scale: AR1 ω 0.09753459 v 0.02 Log scale: AR1 α V Log scale: AR1 φ 0.95 V Degrees of freedom 5.0 V Step 2 Step 4

Step 3: Estimation

Step 4: Graphics



Step 5: Forecasting



Output

| Time Series Lab | - Score Edition | | | - | × |
|-----------------|--------------------------------------------------------------------------------|-------------------|----------|---|---|
| File Info | | | | | |
| | 1t20 t= -1.4//0063/ | | | | ^ |
| | it30 f= -1.47700339 | | | | |
| 1 SI | it40 f= -1.47700295 | | | | |
| | it50 f= -1.47700279 | | | | |
| | it60 f= -1.47700170 | | | | |
| | it70 f= -1.47699985 | | | | |
| Load data | it80 f= -1.47699821 | | | | |
| Louis sais | it85 f= -1.47699821 | | | | |
| | Strong convergence using numerical de Log-likelihood = -6162.036530; T = 42 | erivatives 172 | | | |
| | Ontimized parameter values | | | | |
| | optimized parameter vardes. | | | | |
| Model setup | Parameter type | Value | Free/Fix | | |
| | Log location: AR2 w | 0.0091 | Free | | |
| | Log location: AR2 a | 0.3278 | Free | | |
| | Log location: AR2 ϕ_1 | 1.0073 | Free | | |
| | Log location: AR2 ¢₂ | -0.0113 | Free | | |
| 0110 | Log location: init | 2.8509 | Free | | |
| 0001 | Log location: AR1 (2nd) a | 0.4899 | Free | | |
| 01101- | Log location: AR1 (2nd) φ | 0.8628 | Free | | |
| Estimation | Log location: seasonal a | 0.0000 | Fixed | | |
| | Log location: init seasonal 1 | 0.00/5 | Free | | |
| | Log location: init seasonal 2 | 0.0050 | Free | | |
| | Log location: init seasonal 5 | 0.0000 | Fixed | | |
| | Log location: Init seasonal 4 | 0.0000 | Fixed | | |
| | Log location: p_dummy em | 0.0104 | Free | | |
| | Log location: 6 SPD pag naturn la | 0.0004 | Free | | |
| Craphics | Log location: B WII Close lag1 | -0.3425 | Eree | | |
| Graphics | Log scale: AR1 w | 0.0052 | Free | | |
| | Log scale: AR1 a | 0.1536 | Eree | | |
| | Log scale: AB1 d | 0.9259 | Free | | |
| • A T | Degrees of freedom | 7.4019 | Free | | |
| A T | Estimation process completed in 361 (| 318 seconds | | | |
| | | Seconds | | | |
| Forecasting | S | TATE INFORMA | TIÓN —— | | |
| | Component location | Initial | Time T | | |
| | Mean | 17.4334 | 14.4714 | | |
| | Composite signal | 2.8584 | 2.6722 | | |
| | AR2 | 2.8509 | 2.5987 | | |
| | AR1 (2nd) | 0.0000 | 0.0640 | | |
| | Seasonal | 0.0075 | 0.0050 | | |
| | Хβ | 0.0000 | 0.0044 | | |
| | Component scale | Initial | Time T | | |
| | Standard deviation | 1.2560 | 1.0926 | | |
| | Composite signal | 0.0704 | -0.0689 | | |
| | AR1 | 0.0704 | -0.0689 | | |
| | | | | | ~ |
| | | | | | |